# MATH 547 Modeling and Analysis of Partial Differential Equations Department of Mathematics

Winter Semester 2025

Professor: Vianey Villamizar Office: 318 TMCB Class: 2:00 - 2:50 p.m. MWF 108 TMCB Email: <u>vianey@math.byu.edu</u>

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Office Hours: Monday 3:00-4:00 pm at my Office 318 TMCB Problem Session Friday 3:00-5:00 pm (at XXXX) Text: Applied Partial Differential Equations. Richard Haberman. 5<sup>th</sup> Edition.

### OUTLINE

#### 1. Partial Differential Equation (PDE) Models. Conservation Laws (My Notes).

- 1.1 Incompressible and Inviscid Fluid Equations.
  - 1.1.1 The Continuity Equation
  - 1.1.2 Euler's Equation of Motion
- 1.2 Diffusion Equation
  - 1.2.1 The Heat Equation.
  - 1.2.2 Well-Posedness
- 1.3 Equilibrium Equations
  - 1.3.1 Laplace's Equation.
  - 1.3.2 Integral Identities.
  - 1.3.3 Well-Posedness.
  - 1.3.4 Laplace Equation for Fluid Stream Function (2.5.3 Haberman)

#### 2. Green's Functions for Time-Independent Problems [Chap 9 Haberman]

9.3 Green's Functions of BVP for Ordinary Differential Equations.

9.3.1 One-Dimensional Steady State Heat Equation.

9.3.4 The Dirac Delta Function.

- 9.3.5 Nonhomogeneous Boundary Conditions.
- 9.5 Green's Function for Poisson's Equation.

9.5.1 Introduction.

9.5.2 Multidimensional Dirac Delta Function and Green's Functions.

9.5.3 Green's Function by the Method of Eigenfunctions Expansion and the Fredholm Alternative.

9.5.5 Green's Function for Problems with Nonhomogeneous Boundary Conditions.

9.5.6 Infinite Space Green's Functions.

- 9.5.7 Green's Function for Bounded Domains.
- 9.5.8 Green's Function for a Semi-infinite Plane. Method of Images.
- 9.5.9 Green's Function for a Circle. Method of Images.

## 3. Green's Functions for the Wave Equation [Chap 11 Haberman]

- 11.2 Green's Functions for the Wave Equation
  - 11.2.2 Green's Formula for the Wave Equation.
  - 11.2.3 Reciprocity.
  - 11.2.4 Using the Green's Function.
  - 11.2.5-6 Differential equations for the Green's Function.
  - 11.2.7 Infinite Space Green's Function (1D). d'Alambert's Solution
  - 11.2.8 Infinite Space Green's Function for the Three-Dimensional Wave Equation. Huygens' Principle.
  - 11.2.9 Two-Dimensional Infinite Space Green's Function.

## 4. Green's Functions for the Heat Equation [Chap 11 Haberman]

11.3 Green's Functions for the Heat Equation.

- 11.3.2 Non-Self-Adjoint Nature of the Heat Equation.
- 11.3.3 Green's Formula for the Heat Equation.

11.3.4-5 Adjoint Green's Function. Reciprocity.

11.3.6 Representation of the Solution Using Green's Function

11.3.7 Alternate Differential equations for the Green's Function.

11.3.8-10 Green's Functions for Infinite, Semi-infinite and Finite Domains.

11.3.8 Infinite Space Green's Function for the Three-Dimensional Wave Equation.

# 5. Method of Characteristics [Chap 12 Haberman]

### Linear and Quasilinear Wave Equations

12.2 First Order wave Equation.

- 12.3 One-Dimensional Second Order Wave Equation. Infinite Domain.
- 12.4 Semi-infinite Domain.
- 12.5 Finite-Domain.

12.6 Method of Characteristic for Quasilinear PDEs. Traffic Flow. Shock Waves. Entropy Condition.

### 6. Infinite Domain Problems: [Chap 10 Haberman]

### Fourier Transform Solutions of PDE.

- 10.2 Heat Equation on Infinite Domain.
- 10.3 Fourier Transform Pair.
- 10.4 Fourier Transform and the Heat Equation. Convolution Theorem
- 10.5 Fourier Sine and Cosine Transforms.
- 10.6 Examples Using Transforms.
- 10.7

### **REFERENCE LIST**

- [LS] C. C. Lin and L. A. Segel, *Mathematics Applied to Deterministic Problems in the Natural Sciences*, SIAM, 1988.
- [DF] D. G. Duffy, *Green's Functions with Applications*, Chapman & Hall/CRC, 2001.
- [OHL] J. Ockendon, S. Howinson, and A. Lacey, *Applied Partial Differential Equations, Oxford University Press*, 2003.

- [S] L. E. Segel, *Mathematics Applied to Continuum Mechanics*, Macmillan Publishing Co., Inc, 1977.
- [H] R. Haberman, *Applied Partial Differential Equations*, 4<sup>th</sup> Edition, Pearson Prentice Hall, 2004.
- [G] M. S. Gockenbach, *Partial Differential Equations* SIAM, 2002.
- [DR] L. Dresner, *Applications of Lie's Theory of Ordinary and Partial Differential Equations*, IOP Publishing Ltd, 1999.
- [TS] A. N. Tikhonov and A. A. Samarskii, *Equations of MathematicalPhysics*, Macmillian, 1963.
- [DF2] D. G. Duffy, *Transform Methods for Solving Partial Differential Equations*, Chapman & Hall/CRC, 2004.

#### Homework

Homework will be assigned every week with their corresponding due date (mostly on Monday). Help with the solutions of homework problems will be held during a **weekly review session on Friday evenings**. This is optional but I can anticipate that it will be of great benefit for the students. Discussion of homework assignments is recommended, but you should keep in mind that homework is an individual work. If you can reach to the point where you can do fresh problems without help in all sections, I can anticipate that you will be able to successfully solve all problems in homework and exams.

You should be willing to put in at least two to three hours outside the classroom for each hour of class (The PDE problems normally require a dedicated work. In most cases each problem is different from others). Late homework won't be accepted. To make up for this, your two lowest homework (Book Sections) scores will be dropped.

#### Homework Format and Submission (PLEASE ADHERE TO THE FOLLOWING

**HOMEWORK FORMAT):** Put your name at the top of each sheet. Keep problems in order and label each problem with its corresponding book Section and book number. Place only one problem in any horizontal space; visually separate consecutive problems by drawing a line between them entirely across the page. Preferably, do not answer two different problems in the same page. If the problem has a numerical answer, highlight it in some way. If the answer to a problem involves a sequence of logical steps, set them clearly. Use correct grammar and complete sentences.

You will use Gradescope to submit your homework. If you do not have an account in Gradescope, you may have received an email with indications of your account in Gradescope. If not, you can create your account at **Gradescope**: <u>https://www.gradescope.com</u>

PLEASE, USE THE SAME email ADDRESS IN THE GRADESCOPE MESSAGE THAT YOU RECEIVED. Gradescope will allow you to upload your handwritten answers for your homework. Just follow the Gradescope procedure described in its webpage. You will have until 11:59 pm to complete and upload your written homework for the dates indicated in the homework schedule. Make sure you reserve enough time (20-30 mins.) for the uploading process.

Homework problems to be graded will be chosen among the whole set of problems. Incomplete homework will receive partial credit according to the amount of problems worked out.

**Exams:** The Midterm exams will be based on the material (theory and homework problems) covered until the previous lecture. The Midterm exams will be given in the testing center as follows:

#### Midterm Exams:

- Midterm 1 Tue Feb 11 -- Thu Feb 13 (late day Feb 13)
- Midterm 2 Fri Mar 28 Mon Mar 31 (late day Mar 31) Each midterm will be held in the Testing Center.

They will have a time limit of four (4) hours. **Only basic scientific calculators (no graphic or symbolic ones) will be allowed in the Midterm exams**. The questions will be like those discussed in class, or those assigned as homework, but some of them will require a good understanding of the concepts and techniques. The best way to prepare for it is to go over the homework problems and the examples worked in class (they constitute your **best study guide**) and then try to solve related problems that you haven't seen before.

#### Grading: Grades will be based on cumulative points earned as follows: Homework 40%, Midterms 20% each (2 in total) and Final project 20 %.

At the end of the semester, I will make an average of each one of the above forms of evaluations with their corresponding weights. Then a Gaussian curve, if needed, will help me to determine your final grade. In any event, the Gaussian curve will not hurt your grade. I will guarantee the following letter grades:

	B + = 89-87%,	C+=79-77%,	D + = 69 - 67%,
A = 100-93%,	B = 86-83%,	C = 76-73%,	D = 66-63%, E = 59-0%
A - = 92 - 90%,	B- = 82-80%,	C- = 72-70%,	D- = $62-60\%$ .

Keep in mind that a good grade is the result of a good learning process. All of you can get a good grade by successfully experiencing this learning process.

**Honor Code:** In keeping with the principles of the BYU Honor Code, students are expected to be honest in all of their academic work. Academic honesty means, most fundamentally, that any work you present as your own must in fact be your own work and not that of another. Violations of this principle may result in a failing grade in the course and additional disciplinary action by the university. Students are also expected to adhere to the Dress and Grooming Standards. Adherence demonstrates respect for yourself and others and ensures an effective learning and working environment. It is the university's expectation, and my own expectation in class, that each student will abide by all Honor Code standards. Please call the Honor Code Office at 422-2847 if you have questions about those standards.

**Preventing & Responding to Sexual Misconduct:** Brigham Young University prohibits all forms of sexual harassment—including sexual assault, dating violence, domestic violence, and stalking on the basis of sex— by its personnel and students and in all its education programs or activities. University policy requires all faculty members to promptly report incidents of sexual harassment that come to their attention in any way and encourages reports by students who experience or become aware of sexual harassment. Incidents should be reported to the Title IX Coordinator at <u>t9coordinator@byu.edu</u> or (801) 422-8692 or 1085 WSC. Reports may also be submitted online at <u>https://titleix.byu.edu/report</u> or 1-888-238-1062 (24-hours a day). BYU offers a number of resources and services for those affected by sexual harassment, including the university's confidential Sexual Assault Survivor Advocate. Additional information about sexual harassment, the university's Sexual Harassment Policy, reporting requirements, and resources can be found in the University Catalog, by visiting <u>http://titleix.byu.edu</u>, or by contacting the university's Title IX Coordinator.

**Student Disability:** Brigham Young University is committed to providing a working and learning atmosphere that reasonably accommodates qualified persons with disabilities. A disability is a physical or mental impairment that substantially limits one or more major life activities. Whether an impairment is substantially limiting depends on its nature and severity, its duration or expected duration, and its permanent or expected permanent or long-term impact. Examples include vision or hearing impairments, physical disabilities, chronic illnesses, emotional disorders (e.g., depression, anxiety), learning disorders, and attention disorders (e.g., ADHD). If you have a disability which impairs your ability to complete this course successfully, please contact the University Accessibility Center (UAC), 2170 WSC or 801-422-2767 to request a reasonable accommodation. The UAC can also assess students for learning, attention, and emotional concerns. If you feel you have been unlawfully discriminated against on the basis of disability, please contact the Equal Opportunity Office at 801-422-5895, <u>eo\_manager@byu.edu</u>, or visit <u>https://hrs.byu.edu/equal-opportunity</u> for help.